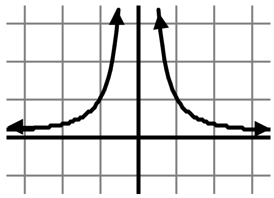
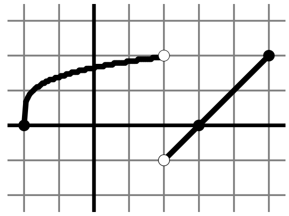
ASSIGNMENT #7

Use these graphs of f' to graph f''.

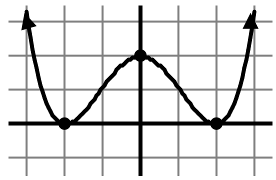
1. f'



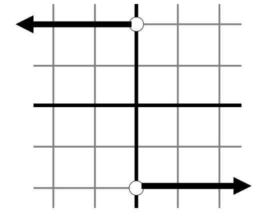
2. f'



3. f'



4. f'

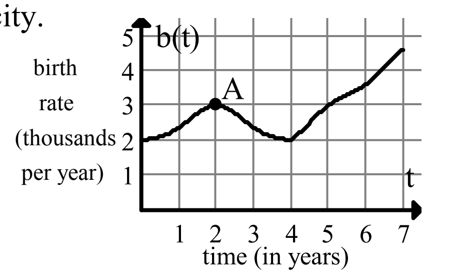


5 & 6. Use the graphs of f' for Problems 1 and 2 above to sketch a possible graph of f.

7. Use the graph of f' in Problem 3 to sketch a graph of f with the starting point $(-2,-2)$.

8. Use the graph of f' in Problem 4 to sketch a continuous graph of f with the starting point $(0,2)$.

9. The graph at the right models the birth rate in a Utah city.



(t = 0 at the beginning of 1990)

t	0	1	2	3	4	5	6	7
b	2	2.3	3	2.2	2	3	3.5	4.5

a. Tell what Point A represents.

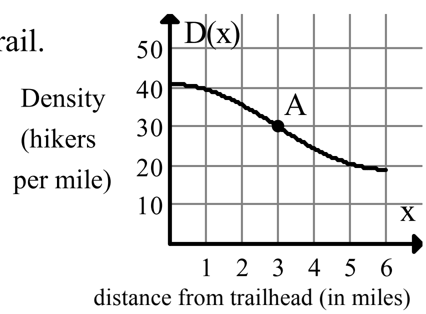
b. Approximate $\int_2^6 b(t)\,dt$ using a midpoint Riemann Sum with 2 equal subintervals.

c. Write a sentence telling what your answer to Part b represents using numbers and units.

d. Use the given information to approximate $b'(5)$. Show your work.

e. Write a sentence telling what your answer to Part d represents using correct units.

10. The graph at the right represents the density of hikers on a trail.



x	0	1	2	3	4	5	6
D	41	40	35	30	25	20	19

a. Tell what Point A represents.

b. $\int_0^3 D(x)\,dx = 110$. Write a sentence with numbers and units stating what this represents.

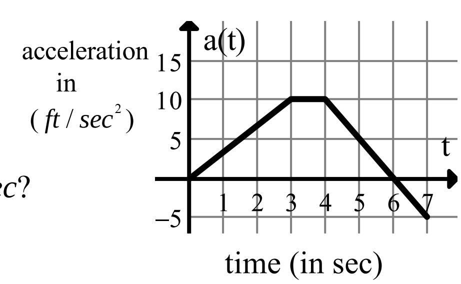
c. If $D'(3) = -6$, use <u>local linearization</u> to approximate the density of hikers 3.1 miles from the trailhead.

d. Write an integral expression for the total number of hikers on the first six miles of the trail.

e. Use the trapezoidal rule with six subdivisions to approximate your integral in Part d.

f. Use your answer from Part e to approximate the average density of hikers in these six miles.

11. The graph at right models acceleration



a. What is the acceleration at $t = 2$ *sec*?

b. When is the acceleration 10 ft/sec^2?

c. What is the minimum acceleration?

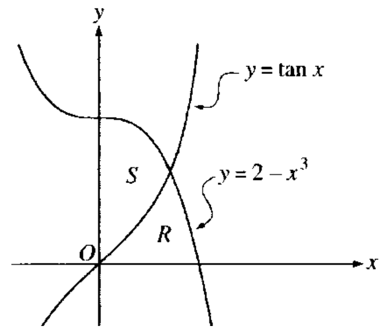
d. If the initial velocity is zero, what is the velocity at $t = 6$ *sec*?

e. If the initial velocity is 20 *ft/sec*, what is the velocity at $t = 6$ *sec*?

f. If the initial velocity is 20 *ft/sec*, what is the velocity at $t = 7$ *sec*?

Calculators may be used on the first three problems only.

12. Let *R* and *S* be the regions in the first quadrant shown in the figure The region *R* is bounded by the *x*-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region *S* is bounded by the *y*-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.



(a) Find the area of *R*.

(b) Find the area of *S*.

(c) Find the volume of the solid generated when *S* is revolved about the *x*-axis.

13. The temperature, in degrees Celsius (°C), of water in the pond is a differentiable function W of time t. The table shows the water temperature as recorded every 3 days over a 15-day period.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

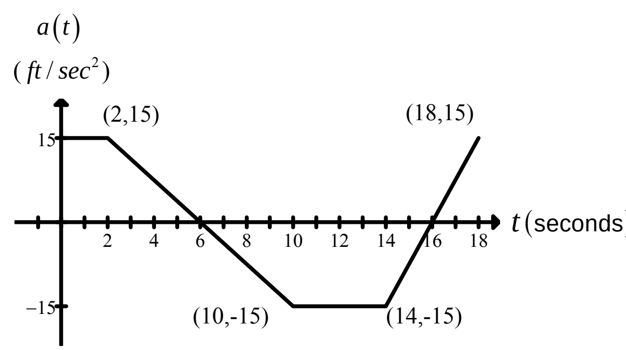
(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using the trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

(c) A student proposes the function P, given by $P(t) = 20 + 10te^{\left(\frac{-t}{3}\right)}$, as a model for the temperature of the water in the pond at time *t*, where *t* is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over time the interval $0 \le t \le 15$ days.

14. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph shown.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what time in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

No calculator is allowed for these problems.

15. Let *h* be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of *h* is given by

$h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all the values of *x* for which the graph of *h* has a horizontal tangent, and determine whether *h* has a local maximum, a local minimum, or neither at each of these values. Justify your answer.

(b) On what intervals, if any, is the graph of *h* concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of *h* at $x = 4$.

(d) Does the line tangent tot the graph of *h* at $x = 4$ lie above or below the graph of *h* for $x > 4$? Why?

16. A cubic polynomial function *f* defined by $f(x) = 4x^3 + ax^2 + bx + k$ where *a, b* and *k* are constants. The function *f* has a local minimum at *x* = -1, and the graph of *f* has a point of inflection *x* = -2.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of *k*?

17. The function *f* is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope of each point (x, y) on the graph is given by $\dfrac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\dfrac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \dfrac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \dfrac{1}{4}$.