You may use a calculator on the entire assignment.

1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

1996 AB 3

2. The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year at t is measured in years from the beginning of 1980.

(a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.

(b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate the units of measure.

(c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

(d) Using correct unites, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

1989 AB 6

3. Oil is being pumped continuously from a certain oil well at a rate of proportional to the amount of oil left in the well, that is, $\frac{dV}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

(a) Write an equation for y, the amount of oil remaining in the well at any time t.

(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining.

(c) In order not to lose money, at what time t should oil no longer be pumped from the well?

1999 AB 3

4. The rate at which water flows out a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured ever 3 hours for a 24-hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

(a) Use a midpoint Riemann sum with 4 subdivision of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

(c) The rate of water flow, $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate the units of measure.

1997 AB 6

5. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dy}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

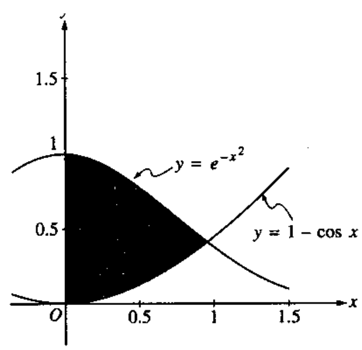
AB 1/BC 1 Calculator Allowed

6. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

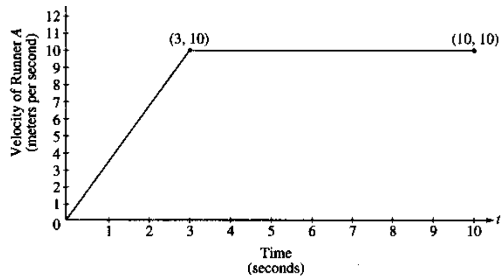
(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



AB 2/BC 2 Calculator allowed

7. Two runners *A* and *B*, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph at the right which consists of two line segments, shows the velocity, in meters per second, of Runner *A*. the velocity, in meters per second, of Runner *B* is given by the function *v* defined by $v(t) = \dfrac{24t}{2t+3}$



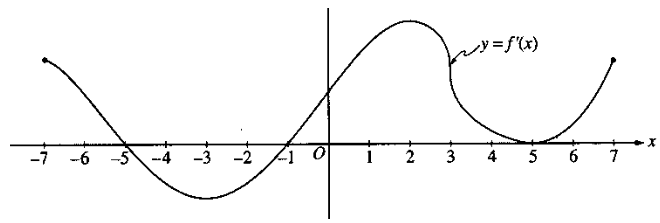
(a) Find the velocity of Runner *A* and the velocity of Runner *B* at time $t = 2$ seconds. Indicate the units of measure.

(b) Find the acceleration of Runner *A* and the acceleration of Runner *B* at time $t = 2$ seconds. Indicate the units of measure.

(c) Find the total distance run by Runner *A* and the total distance run by Runner *B* over the time interval $0 \le t \le 10$ seconds. Indicate the units of measure.

AB 3

8. The figure at the right shows the graph of f', the derivative of the function *f*, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



(a) Find all the values of *x*, $-7 < x < 7$, at which *f* attains a relative minimum. Justify your answer.

(b) Find all the values of *x*, $-7 < x < 7$, at which *f* attains a relative maximum. Justify your answer.

(c) Find all the values of *x*, $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what values of *x*, $-7 \le x \le 7$, does *f* attain an absolute maximum? Justify your answer.

AB-4 Calculator allowed

9. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ and $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time *t*.

(d) At what time *t*, $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

AB 5/BC 5

10. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all the points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

AB 6

11. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).