

Review Assignment #5

$$\textcircled{1} \quad g(x) = \int_2^x f(t) dt$$

$$\text{a) } g(3) = \int_2^3 f(t) dt = \frac{1}{2}(1)(4+2) = 3$$

$$g'(x) = \frac{d}{dx} \int_2^x f(t) dt = f(x) \rightarrow g'(3) = f(3) = 2$$

$$g''(x) = f'(x) \rightarrow g''(3) = \underbrace{f'(3)}_{\text{slope at } 3} = -2$$

$$\text{b) average rate of change} \rightarrow \frac{g(3) - g(0)}{3 - 0}$$

$$g(3) = \int_2^3 f(t) dt = 3 \quad \frac{3+4}{3} = \frac{7}{3}$$

$$g(0) = \int_2^0 f(t) dt = - \int_0^2 f(t) dt = -4$$

$$\text{c) } g'(x) = f(x) \rightarrow g'(c) = f(c)$$

$$\text{where does } \underbrace{f(c)}_{y\text{-value}} = \frac{7}{3} = 2\frac{1}{3}$$

There will be 2 c-values where $f(c) = \frac{7}{3}$.

$$\text{d) POF} \rightarrow f''(x) > 0 \rightarrow f''(x) < 0$$

or

$$f''(x) < 0 \rightarrow f''(x) > 0$$

$$g''(x) = f'(x)$$

where does the slope of f change signs

$$\begin{array}{l} x=2 \\ x=5 \end{array}$$

$$\textcircled{2} \quad \text{a) } x^2 = 4 \quad \underset{x=\pm 2}{2 \int_0^2 [4-x^2] dx} = 2 \left[4x - \frac{1}{3}x^3 \right]_0^2 = 2 \left[(4(2) - \frac{1}{3}(2)^3) - 0 \right] \\ 2 \left[\frac{24}{3} - \frac{8}{3} \right] = 2 \left(\frac{16}{3} \right) = \frac{32}{3} \text{ u}^2$$

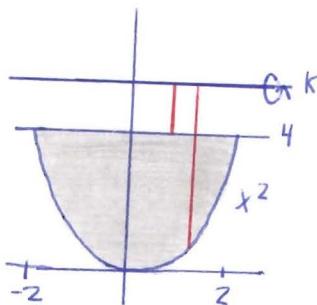
b) washer

$$\pi \int_a^b R^2 - r^2 \rightarrow 2\pi \int_0^2 4^2 - (x^2)^2 dx \rightarrow 2\pi \int_0^2 16 - x^4 dx$$

 over

$$\begin{aligned}
 &= 2\pi \left[16x - \frac{1}{5}x^5 \right]_0^2 = 2\pi \left[(16(2) - \frac{1}{5}(2)^5) - 0 \right] \\
 &= 2\pi (32 - \frac{32}{5}) \\
 &= 2\pi \left(\frac{160}{5} - \frac{32}{5} \right) = \frac{256\pi}{5}
 \end{aligned}$$

c)



washer

$$R: k-x^2$$

$$r: k-4$$

$$2\pi \int_0^2 (k-x^2)^2 - (k-4)^2 dx = \frac{256\pi}{5}$$

③ $0 \leq t \leq 5$ $v(t) = 3(t-1)(t-3)$ $x(2) = 0$

a) minimum accel $\rightarrow f''(x) < 0 \rightarrow f''(x) > 0$

b	$t-2$	$+/-$
$6t-12=0$	$t=2$	$-/-$
		$\textcircled{-} \quad \textcircled{+}$

endpoints
 $(0, -12)$
 $(5, 18)$

critical pts.
 $(2, 0)$

minimum acceleration
happens at $(0, -12)$

b) $v(t) = 3(t-1)(t-3) = 0$ $x(t) = t^3 - 6t^2 + 9t - 2$

$$t=1 \quad t=3$$

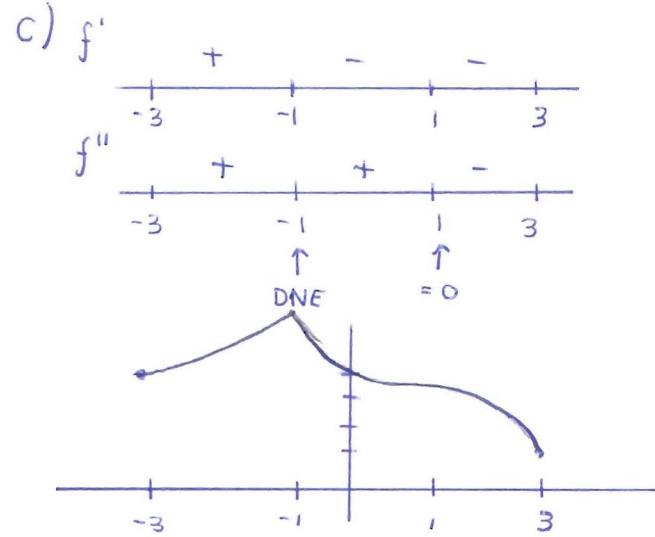
$$\begin{aligned}
 x(t) &= \int 3t^2 - 12t + 9 \\
 &= t^3 - 6t^2 + 9t + C \\
 0 &= 2^3 - 6(2)^2 + 9(2) + C \\
 0 &= 8 - 24 + 18 + C \\
 C &= -2
 \end{aligned}$$

$$\begin{aligned}
 (0, -2) \\
 (1, 2) \\
 (3, -2) \\
 (5, 18)
 \end{aligned}
 \}^4 \quad 28 \text{ units}$$

c) $\frac{1}{5-0} \int_0^5 3t^2 - 12t + 9 = 4 \text{ u/time}$

④ a) abs max at $x = -1$
 abs min at $x = 3$

b) $f''(x) > 0 \rightarrow < 0$ $x = 1$
 $f''(x) < 0 \rightarrow > 0$



⑤ $f(x) = x^3 - 6x^2 + P$

a) $f'(x) = 3x^2 - 12x$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0 \quad x = 4$

$3x \quad - \quad + \quad +$

$x-4 \quad - \quad - \quad +$

$\begin{array}{c} \oplus 0 \quad \ominus 4 \\ \uparrow \max \quad \uparrow \min \end{array}$

$\max \text{ at } (0, P)$

$\min \text{ at } (4, -32+P)$

b) $\frac{1}{2+1} \int_{-1}^2 (x^3 - 6x^2 + P) dx = 1 \rightarrow \int_{-1}^2 (x^3 - 6x^2 + P) dx = 3$

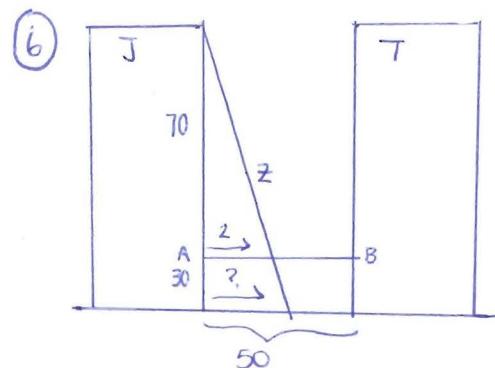
$\left. \frac{1}{4}x^4 - 2x^3 + Px \right|_{-1}^2 = 3$

$3P = \frac{69}{4}$

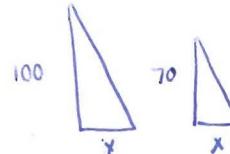
$P = \frac{23}{4}$

$(4 - 16 + 2P) - (-1/4 + 2 - P) = 3$

$-12 + 2P - 9/4 + P = 3$



a) similar \triangle 's



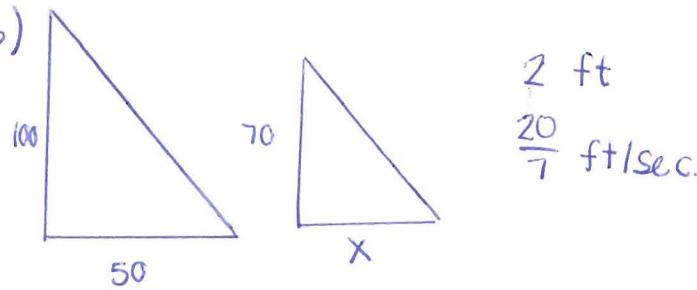
$\frac{x}{70} = \frac{y}{100}$

$70y = 100x$

$70 \frac{dy}{dt} = 100 \frac{dx}{dt}$

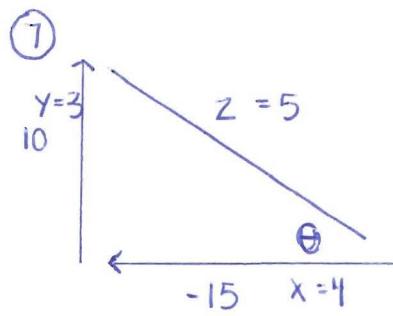
$70 \frac{dy}{dt} = 100 * 2$

$\frac{dy}{dt} = \frac{20}{7} \text{ ft./sec.}$



$$\frac{70}{x} = \frac{100}{50} \quad 100x = 3500$$

$$x = 35 \text{ ft.}$$



a) $x^2 + y^2 = z^2$

$$4^2 + 3^2 = z^2$$

$$z = 5 \text{ km}$$

b) $x^2 + y^2 = z^2$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(4)(-15) + 2(3)(10) = 2(5) \frac{dz}{dt}$$

$$-120 + 60 = 10 \frac{dz}{dt}$$

$$\frac{dz}{dt} = -6 \text{ km/hr}$$

c) $\sin \theta = \frac{y}{z}$

$$u = y \quad u' = \frac{dy}{dt}$$

$$v = z \quad v' = \frac{dz}{dt}$$

$$\cos \theta = \frac{x}{z}$$

$$\cos \theta \frac{d\theta}{dt} = \frac{z \frac{dy}{dt} - y \frac{dz}{dt}}{z^2}$$

$$\frac{4}{5} \left(\frac{d\theta}{dt} \right) = \frac{5(10) - 3(-6)}{5^2}$$

$$= \frac{4}{5} \frac{d\theta}{dt} = \frac{50 + 18}{25}$$

$$\frac{d\theta}{dt} = \frac{\frac{68}{25}}{\frac{5}{4}} = \frac{17}{5} \text{ radians/hr.}$$

$$⑧ V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = 28\pi$$

$$r = 3$$

$$V = 12\pi$$

$$\frac{dr}{dt} = \frac{1}{2}$$

a) Area of base is a circle. $\frac{dA}{dt} = ?$

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(3)\left(\frac{1}{2}\right) = 3\pi \text{ cm}^2/\text{sec.}$$

b) $\frac{dh}{dt} = ?$ $12\pi = \frac{1}{3}\pi (3)^2 h$
 $h = 4$

$$V = \frac{1}{3}\pi r^2 h \rightarrow V = \frac{1}{3}Ah \quad u = \frac{1}{3}A \quad u' = \frac{1}{3}\frac{dA}{dt}$$

$$v = h \quad v' = \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{3}A \frac{dh}{dt} + \frac{1}{3}h \frac{dA}{dt}$$

$$28\pi = \frac{1}{3}(9\pi) \frac{dh}{dt} + \frac{1}{3}(4)(3\pi)$$

$$24\pi = 3\pi \frac{dh}{dt} \rightarrow \frac{dh}{dt} = 8 \text{ cm/sec.}$$

$$\begin{cases} A = \pi r^2 \\ A = 9\pi \\ \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \\ = 2\pi(3)\left(\frac{1}{2}\right) \\ = 3\pi \end{cases}$$

c) $r = 3$

$$\frac{dA}{dh} = ?$$

$$h = 4$$

$$\frac{dA}{dh} = \frac{\frac{dA}{dt}}{\frac{dh}{dt}} = \frac{3\pi}{8} \text{ cm/sec}$$

$$⑨ h = 12$$

$$d = 8$$

$$A = 400\pi$$

$$\frac{dh}{dt} = h - 12$$

a) $V = \frac{1}{3}\pi r^2 h$

$$\frac{r}{h} = \frac{4}{12}$$

$$12r = 4h \rightarrow r = \frac{1}{3}h$$

$$V = \frac{1}{3}\pi \left(\frac{1}{3}h\right)^2 h \rightarrow V = \frac{1}{27}\pi h^3$$

b) $\frac{dV}{dt} = ?$

$$V = \frac{1}{27}\pi h^3$$

$$\rightarrow \frac{dV}{dt} = \frac{1}{9}\pi (3)^2 (3-12)$$

$$h = 3$$

$$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = -9\pi \text{ ft}^3/\text{min}$$

$$c) V = \pi r^2 y$$

\downarrow constant

$$A = \pi r^2$$

$$400\pi = \pi r^2$$

$$r^2 = 400$$

volume cone change: -9π
 \hookrightarrow volume cylinder change: 9π

$$V = 400\pi y \quad \frac{dV}{dt} = 400\pi \frac{dy}{dt} \rightarrow 9\pi = 400\pi \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{9}{400} \text{ ft/min}$$

$$⑩ y = \frac{9}{625}x^4 \quad 0 < x < 5$$

a) shells $\rightarrow 2\pi \int_a^b (\underbrace{\text{radius}}_x) (\underbrace{\text{height}}_{9 - (\frac{9}{625}x^4)}) dx$

$$2\pi \int_0^5 x \left(9 - \frac{9}{625}x^4\right) dx = 471.238 \text{ ft}^3/\text{min}$$

b) volume = rate * time \rightarrow time = $\frac{\text{volume}}{\text{rate}} = \frac{471.238}{8} \approx 59 \text{ minutes}$

given in problem: rate is $8 \text{ ft}^3/\text{min}$ ($\frac{dv}{dt}$)

c) h: depth

$\hookrightarrow h = y$ - axis \Rightarrow have to integrate w/ horizontal cross sections

$$y = \frac{9}{625}x^4 \rightarrow x = \sqrt[4]{\frac{625}{9}y}$$

disks $V = \pi \int_0^h \sqrt[4]{\frac{25}{3}y} dy \rightarrow \frac{dV}{dt} = \pi \sqrt[4]{\frac{25}{3}h} \frac{dh}{dt}$

$$8 = \pi \sqrt[4]{\frac{25}{3}(B)} \frac{dh}{dt}$$

$$\frac{dh}{dt} = .152 \text{ ft/min}$$