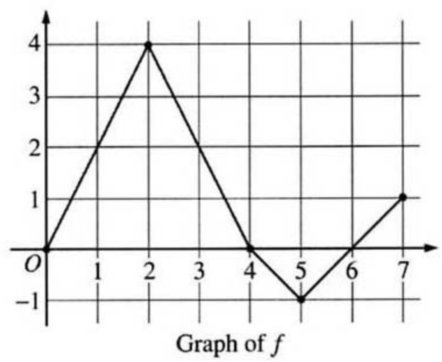
2003 AB 5 (Form B)



1. Let f be the function defined on the closed interval [0, 7]. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

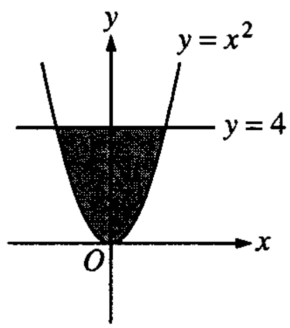
(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

1999 AB 2 No Calculator



2. The shaded region R is bounded by the graph of $y = x^2$, and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x-axis.

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

1992 AB 2 Calculator Allowed

3. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time t = 2, the position of the particle is $x(2) = 0$.

   (a) Find the minimum acceleration of the particle.

   (b) Find the total distance traveled by the particle.

   (c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

1984 AB 4—BC 3

4. A function f is continuous on the closed interval [-3, 3] such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

   (a) What are the x-coordinates of all the absolute maximum and absolute minimum points of f on the interval [-3, 3]? Justify your answer.

   (b) What are the x-coordinates of all points of inflection of f on the interval [-3, 3]? Justify your answer.

   (c) On the axis provided, sketch a graph that satisfies the given properties of f.

1997 AB 4 Calculator Allowed

5. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

   (a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Justify your answer.

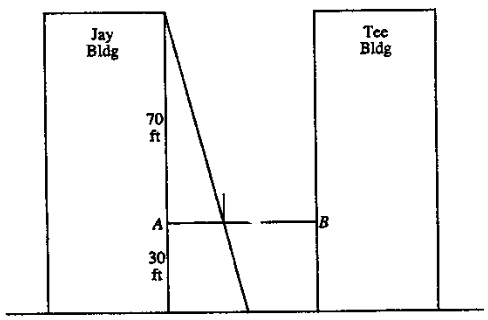
   (b) Find the value of p such that the average value of f over the closed interval [-1, 2] is 1.

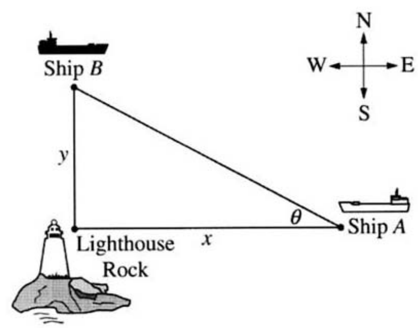
1991 AB 6 Calculator Allowed

6. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point *A* to point *B*, is illuminated by the spotlight 70 feet above point *A*, as shown in the diagram.

(a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

(b) How far from point *A* is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)





2002 AB 6 (Form B) No calculator

7. Ship *A* is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship *B* is traveling due north away from Lighthouse Rock at a speed of 10km/hr. Let x be the distance between Ship *A* and Lighthouse Rock at time t, and let y be the distance between Ship *B* and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship *A* and Ship *B* when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change in θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

1984 AB 5 Calculator allowed

8. The volume V of a cone ($V = \frac{1}{3}\pi r^2 h$) is increasing at the rate of 28π cubic cm per second. At the instant when the radius r of the cone is 3 cm, its volume is 12π cubic cm and the radius is increasing at $\frac{1}{2}$ cm per second.

(a) At the instant when the radius of the cone is 3 cm, what is the rate of change of the area of its base?

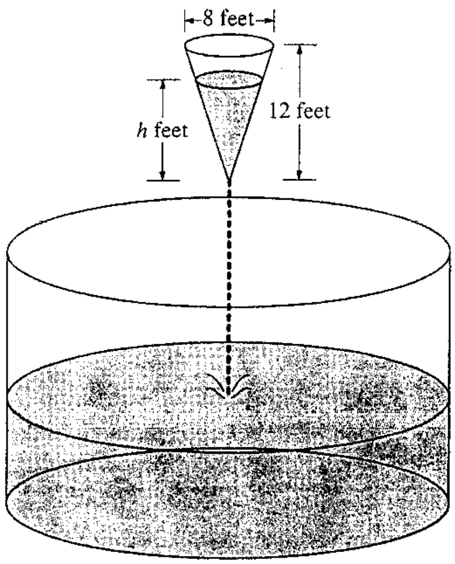
(b) At the instant when the radius of the cone is 3 cm, what is rate of change of its height h?

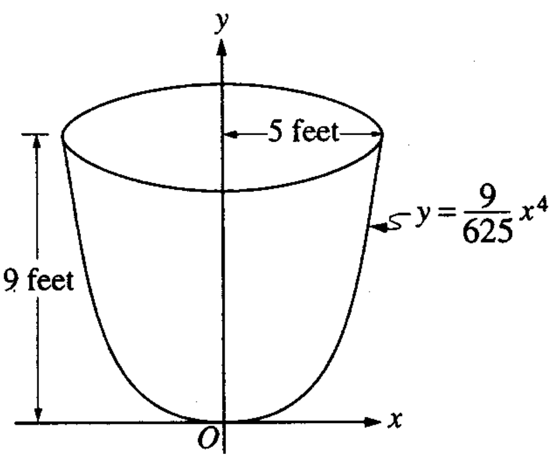
(c) At the instant when the radius if the cone is 3 cm, what is the instantaneous rate of change of the area of its base with respect to its height h?

1995 AB 5 Calculator allowed

9. As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of (h – 12) feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.
(b) At what rate is the volume of water in the conical tank changing when h = 3? Indicate units of measure.
(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when h = 3? Indicate units of measure.





1996 AB 5 Calculator Allowed

10. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

(a) Find the volume of the tank. Indicate units of measure.
(b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?
(c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when h = 4? Indicate units of measure.