

Review Assignment #4

① $y + \cos y = x + 1 \quad 0 \leq y \leq 2\pi$

a) $\frac{dy}{dx} - \sin y \frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{1-\sin y}$$

b) $1 - \sin y = 0$

$$\sin y = 1$$

$$y = \frac{\pi}{2}$$

$$\frac{\pi}{2} + \cos\left(\frac{\pi}{2}\right) = x + 1$$

$$x = 1 - \frac{\pi}{2}$$

c) $u = 1 \quad u' = 0$

$$v = 1 - \sin y \quad v' = -\cos y \frac{dy}{dx} \\ = \frac{-\cos y}{1 - \sin y}$$

$$= \frac{0 + \frac{\cos y}{1 - \sin y}}{(1 - \sin y)^2} = \frac{\cos y}{(1 - \sin y)^3}$$

② $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gal/day $t=0 \rightarrow 50$ gal. 40 gal \rightarrow safe

a) increasing \rightarrow + slope

$$P'(9) = 1 - 3e^{-0.2\sqrt{9}} \\ = -0.646$$

No. slope/derivative $\partial t=9$
is negative.

b) minimum $\rightarrow f'(x) < 0$ to $f'(x) > 0$

$$t = 30.173 \text{ days}$$

c) $\int_0^{30.173} 1 - 3e^{-0.2\sqrt{t}} dt + 50$ $= 35.104$ gal. Yes, the lake is safe.
 ↑ initial amount

d)

Point	Slope	$y - 50 = -2(x - 0)$	$40 = -2x + 50$
$(0, 50)$	$\partial t=0$ $P'(0) = 1 - 3e^0$ $= -2$	$y = -2x + 50$	$-10 = -2x$

$$x = 5 \text{ days.}$$

$$\textcircled{3} \quad t \geq 0 \quad v(t) = -1 + e^{1-t}$$

$$\text{a)} \quad v'(t) = a(t) = e^{1-t} * (-1)$$

$$a(3) = -e^{1-3} \\ = -e^{-2}$$

b) increasing speed

↳ same velocity & acceleration
Sign (+ and + or - and -)

② $t=3$ $a(t)$ is neg.
③ $t=3$ $v(t)$ is neg.

Speed is increasing.

$$\text{c)} \quad v(t) = 0$$

$$-1 + e^{1-t} = 0$$

$$\ln e^{1-t} = \ln 1$$

$$1-t = 0$$

$$t=1$$

$$\text{d)} \quad \int_0^1 -1 + e^{1-t} dt + \int_1^3 -1 + e^{1-t} dt$$

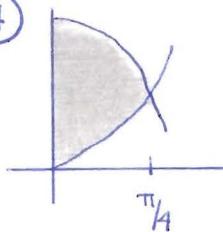
$$= e + e^{-2} - 1 \quad u^2$$

$$u = 1-t$$

$$du = -dt$$

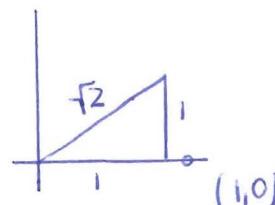
$$-du = dt$$

\textcircled{4}



$$\text{a)} \quad \int_0^{\pi/4} \cos x - \sin x \, dx$$

$$\sin x + \cos x \Big|_0^{\pi/4}$$



$$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right] - [0 + 1]$$

$$= \frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}-2}{2} = \sqrt{2}-1$$

$$\text{b)} \quad R: \cos x - 0 \quad \pi \int_0^{\pi/4} \cos^2 x - \sin^2 x \, dx = 1.571 \text{ u}^3$$

$$r: \sin x - 0$$

$$\text{c)} \quad A = b * h$$

\uparrow

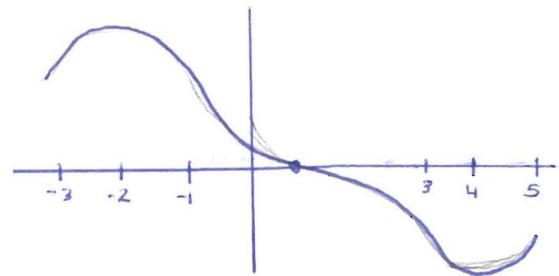
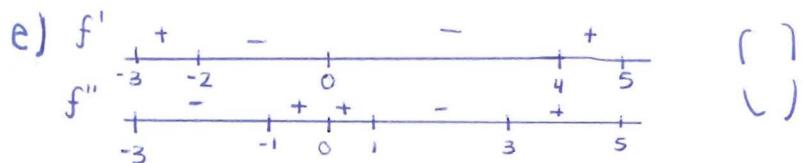
$$(cos x - sin x)^2$$

$$\int_0^{\pi/4} (\cos x - \sin x)^2 \, dx = .285 \text{ u}^3$$

⑤ a) maximum $f'(x) > 0$ to $f'(x) < 0$ (above to below)
 $x = -2$ it is an open interval so no need to look at end points.

b) maximum $f'(x) < 0$ to $f'(x) > 0$ (below to above)
 $x = 4$

c) concave up $f''(x) > 0$ (positive slope on $f'(x)$)
 $(-1, 1)$ and $(3, 5)$



⑥ $\underbrace{\int_a^b f(x)}_{a+2b} + \underbrace{\int_a^b 5}_{5x|_a^b}$ C

$$a+2b + 5b - 5a \\ -4a + 7b$$

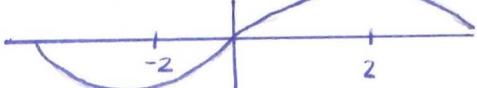
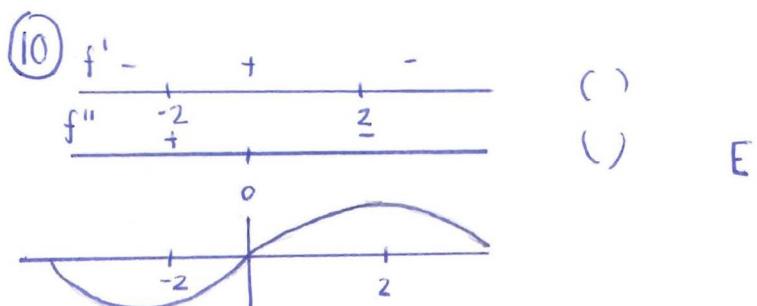
⑦ $f(x) = -x^3 + x + \frac{1}{x}$
 $f'(x) = -3x^2 + 1 - \frac{1}{x^2} \Big|_{x=-1}$ D

$$f'(-1) = -3(-1)^2 + 1 - \frac{1}{(-1)^2} \\ = -3 + 1 - 1 \\ = -3$$

⑧ $v(t) = 0$
 $\text{at } t=6$ C

⑨ $\int_0^8 |v(t)| dt = \cancel{\frac{1}{2}(2)(3)} + 2(3) \\ + \cancel{\frac{1}{2}(2)(3)} + \cancel{\frac{1}{2}(2)(1)}$

$$= 3 + 6 + 3 + 1 \\ = 13 \quad B$$



$$\textcircled{11} \quad f'(x) = \frac{|4-x^2|}{x-2} < 0$$

$$|4-x^2|=0$$

$$x-2=0$$

$$|4-x^2| + + + +$$

$$4-x^2=0$$

$$x-2 \quad - - - +$$

$$x^2=4$$

$$x=\pm 2$$

$$x=2$$

$$(-\infty, 2)$$

A

$$\textcircled{12} \quad f(3)=2 \quad f'(3)=5$$

$$(3, 2) \quad m=5$$

$$y-2=5(x-3)$$

$$y-2=5x-15$$

$$y=5x-13$$

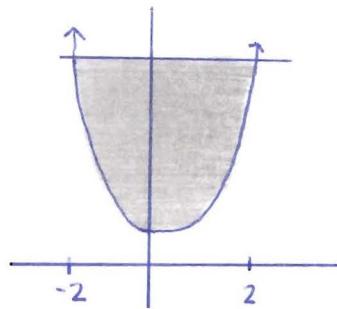
approx a zero

$$0=5x-13$$

$$x=\frac{13}{5}$$

$$=2\frac{3}{5}$$

\textcircled{13}



$$\begin{aligned} x^2+1 &= 5 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

$$\begin{aligned} 2 \int_0^2 5 - (x^2+1) dx &= 2 \int_0^2 4 - x^2 dx \\ &= 2 \left[4x - \frac{1}{3}x^3 \right]_0^2 = 2 \left(\frac{24}{3} - \frac{8}{3} \right) \\ &= 2 \left[(8 - 8/3) - (0) \right] = 2 \left(\frac{16}{3} \right) = \frac{32}{3} \end{aligned}$$

D

$$\textcircled{14} \quad \int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx = \int_0^{\pi/4} e^{\tan x} * \sec^2 x dx$$

$$u = \tan x$$

$$du = \sec^2 x$$

$$\begin{aligned} \int e^u du &= e^u = e^{\tan x} \Big|_0^{\pi/4} = [e^{\tan \pi/4} - e^{\tan 0}] \\ &= e^1 - e^0 = e - 1 \end{aligned}$$

TOA $\neq \frac{\sin}{\cos}$

C

$$\textcircled{15} \quad \frac{1}{5+3} \int_3^5 \cos x dx = \frac{1}{8} \left[\sin x \right]_3^5 = \frac{\sin(5) - \sin(3)}{8}$$

A

$$\textcircled{16} \lim_{x \rightarrow 1} \frac{x}{\ln(x)} = \frac{1}{\ln(1)} = \frac{1}{0} = \text{DNE}$$

E

$$\textcircled{17} f(x) = (x^2 - 3) e^{-x} \quad f'(x) > 0$$

$$u = x^2 - 3 \quad u' = 2x$$

$$v = e^{-x} \quad v' = -e^{-x}$$

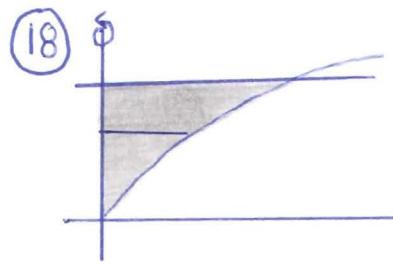
$$-e^{-x}(x^2 - 3) + e^{-x}(2x) = 0$$

$$\begin{aligned} -e^{-x}(x^2 - 3 - 2x) &= 0 \\ -e^{-x} &= 0 \quad x^2 - 2x - 3 = 0 \\ \hookrightarrow \text{never} & \quad (x-3)(x+1) = 0 \\ x = -1 & \quad x = 3 \end{aligned}$$

D

$$\begin{array}{ccccc} -e^{-x} & - & - & - & - \\ x-3 & - & - & + & + \\ x+1 & - & + & + & + \\ \hline - & -1 & + & 3 & - \end{array}$$

$$-1 < x < 3$$



$$\begin{aligned} \textcircled{18} \quad x = y^2 \quad \pi \int_0^2 (y^2)^2 dy &= \pi \int_0^2 y^4 dy = \pi \left[\frac{1}{5} y^5 \right]_0^2 \\ &= \pi \left[\frac{32}{5} - 0 \right] = \frac{32}{5} \pi \end{aligned}$$

A

\textcircled{19} Width of each rectangle is $\frac{1}{50}$. So if there are 50 rectangles
that means $\int_0^1 \sqrt{x}$.

$$\textcircled{20} \quad y = x^3 + 6x^2 + 7x - 2 \cos x$$

$$y' = 3x^2 + 12x + 7 + 2 \sin x$$

$$y'' = 6x + 12 + 2 \cos x$$

$$6x + 12 + 2 \cos x = 0 \quad x = -1.894$$

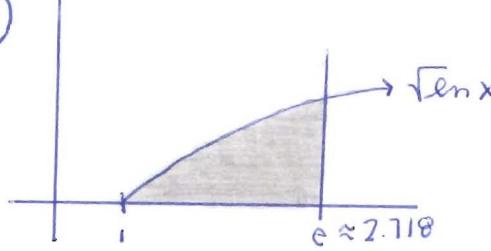
B

D

$$\begin{aligned} \textcircled{21} \quad \int_1^3 f(x) dx &= \int_1^3 F'(x) dx = F(x) \Big|_1^3 = F(3) - F(1) = 2.3 \\ &\quad \underbrace{F(1) - F(0)}_2 = 2 \quad \} 4.3 \\ &\quad \text{area from} \\ &\quad 0 \text{ to } 1 \end{aligned}$$

D

(22)



$$A = b * h$$

$\uparrow \quad \uparrow$
 $\sqrt{\ln x} * \sqrt{\ln x} = \ln x$

$$\int_1^e \ln(x) dx = 1.000 \text{ u}^3$$

C

$$(23) f(x) = \sqrt{x} \rightarrow f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(c) = 2f'(1)$$

$$\frac{1}{2\sqrt{c}} = \frac{1}{2\sqrt{1}} \rightarrow \frac{1}{2\sqrt{c}} = 1 \rightarrow 2\sqrt{c} = 1 \rightarrow \sqrt{c} = \frac{1}{2} \rightarrow c = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

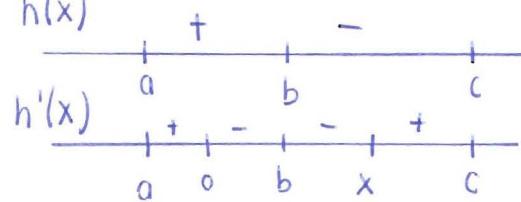
A

$$(24) t \geq 0 \quad a(t) = t + \sin t \quad v(0) = -2$$

$$\begin{aligned} \int t + \sin t dt &= \frac{1}{2}t^2 - \cos(t) + C & \frac{1}{2}t^2 - \cos(t) &= 1 \\ -2 &= \frac{1}{2}(0)^2 - \cancel{\cos'(0)} + C & t &= 1.478 \\ -2 &= -1 + C \\ C &= -1 \end{aligned}$$

B

$$(25) f(x) = \int_a^x h(t) dt \quad h(x)$$



max $\omega x=b$
 POI $\omega x=0 (+ \rightarrow -)$
 $x=x (- \rightarrow +)$

E

$$\begin{aligned} I_1 &= \frac{1}{2} \left(\frac{1}{2}\right) (3+3) = 6/4 \\ I_2 &= \frac{1}{2} \left(\frac{1}{2}\right) (3+5) = 8/4 \\ I_3 &= \frac{1}{2} \left(\frac{1}{2}\right) (5+8) = 13/4 \\ I_4 &= \frac{1}{2} \left(\frac{1}{2}\right) (8+13) = 21/4 \end{aligned} \quad \left. \right\} 48/4 = 12 \text{ u}^2$$

B