1992 AB 4 No Calculator

1. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

2002 AB 2 (Form B) Calculator Allowed

2. The number of gallons, $P(t)$ of a pollutant in a lake changes at a rate of $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of pollutant in the lake a time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at a minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator used the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

2003 AB 4 (Form B) No Calculator

3. A particle moves along the x-axis with velocity at time $t \ge 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

1991 BC 3

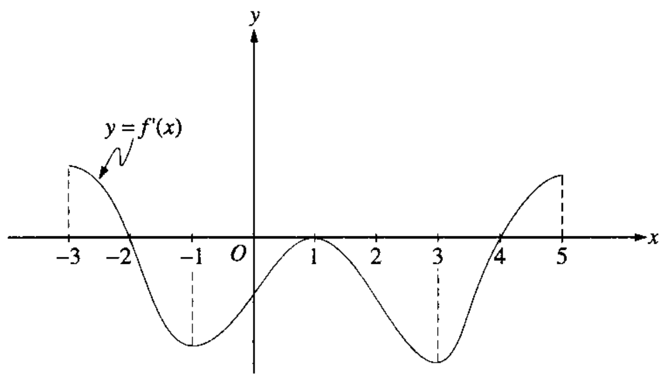
4. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = \sin x$ and $y = \cos x$.

(a) Find the area of R. (without calculator)

(b) Find the volume of the solid generated when R is revolved about the x-axis. (with calculator)

(c) Find the volume of the solid whose base is R and whose cross sections cut by planes perpendicular to the x-axis are squares. (with calculator).

1996 AB 1



5. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?

(b) For what values of x does f have a relative minimum? Why?

(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(e) Suppose that $f(1) = 0$. Draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

DO NOT USE A CALCULATOR ON THIS PORTION.

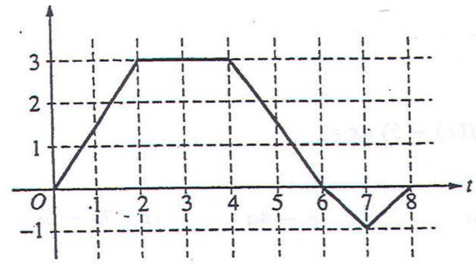
6. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x) + 5)\,dx =$

(A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

7. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) -1 (D) -3 (E) -5

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

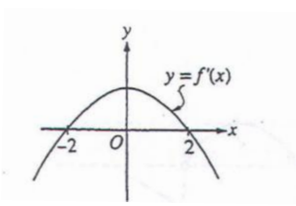
8. At what value of t does the bug change direction?

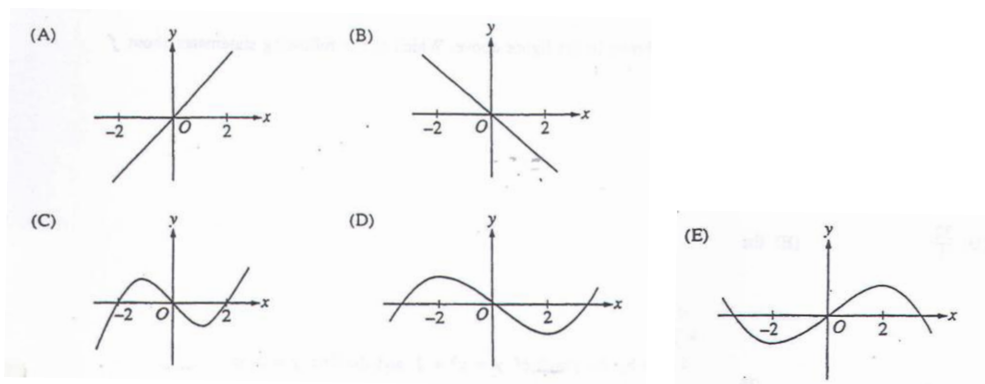
(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

10. The graph of the derivative of f is shown in the figure below. Which of the following could be the graph of f?





11. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

12. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find the approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

13. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

14. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) e - 1 (D) e (E) e + 1

15. The average value of cos x on the interval [-3, 5] is

(A) $\frac{\sin 5-\sin 3}{8}$ (B) $\frac{\sin 5-\sin 3}{2}$ (C) $\frac{\sin 3-\sin 5}{2}$
(D) $\frac{\sin 3+\sin 5}{2}$ (E) $\frac{\sin 3+\sin 5}{8}$

16. $\lim_{x\to 1}\frac{x}{\ln x}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

17. What are all the values of x for which the function f defined by $f(x)=\left(x^2-3\right)e^{-x}$ is increasing?

(A) There are no such values of x (B) $x<-1$ *and* $x>3$ (C) $-3<x<1$
(D) $-1<x<3$ (E) All values of x

18. If the region enclosed by the y-axis, the line y = 2, and the curve $y=\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

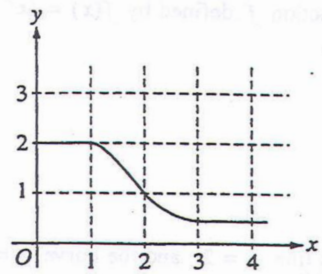
19. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+...+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1\sqrt{\frac{x}{50}}\,dx$ (B) $\int_0^1\sqrt{x}\,dx$ (C) $\frac{1}{50}\int_0^1\sqrt{\frac{x}{50}}\,dx$
(D) $\frac{1}{50}\int_0^1\sqrt{x}\,dx$ (E) $\frac{1}{50}\int_0^{50}\sqrt{x}\,dx$

YOU MAY USE A CALCULATOR ON THE REMAINING PROBLEMS. It will not be needed on all problems.

20. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33



21. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

22. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

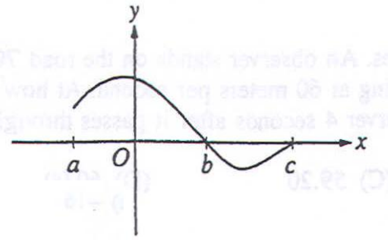
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}\left(e^3 - 1\right)$

23. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

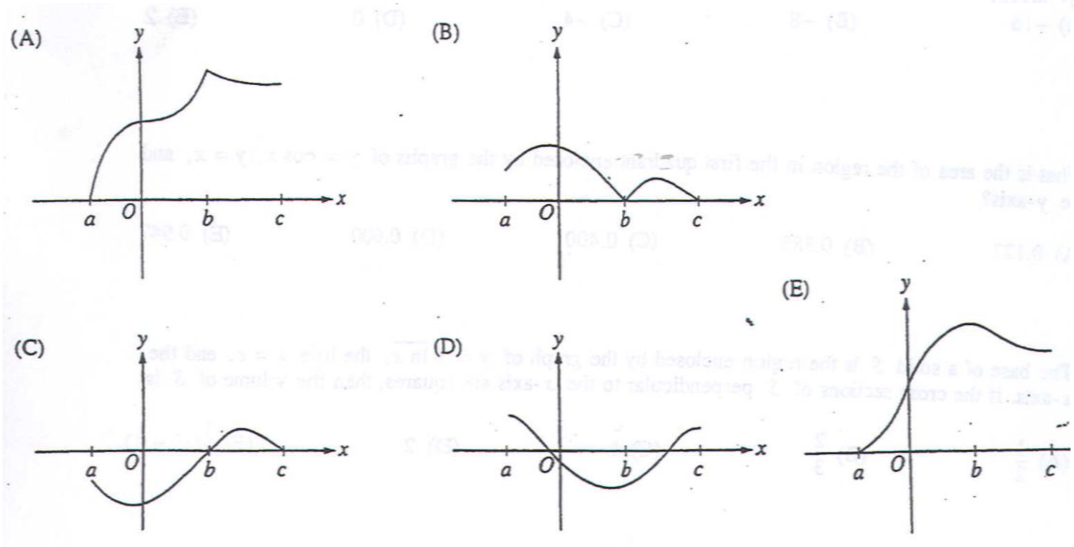
(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

24. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14



25. Let $f(x) = \int_a^x h(t)\,dt$, where h has the graph shown at the right.
Which of the following could be the graph of f?



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

26. A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32