

Review Assignment #3

① $f(x) = x^4 - 3x^2 + 2$

a) $x^4 - 3x^2 + 2 = 0$

$$(x^2 - 2)(x^2 - 1) = 0$$

$$x = \pm\sqrt{2}, \pm 1$$

b) Point

$$x = 1$$

$$y = 1^4 - 3(1)^2 + 2$$

$$= 1 - 3 + 2$$

$$= 0$$

$$(1, 0)$$

slope

$$4x^3 - 6x \Big|_{x=1}$$

$$= 4 - 6$$

$$= -2$$

$$y = -2x - 2$$

c) derivative = m of line

$$4x^3 - 6x = -2$$

$$4x^3 - 6x + 2 = 0$$

$$2(2x^3 - 3x + 1) = 0$$

$$x = -1.366, .366, 1$$

② $x(t) = \sin(\pi t^2) \quad -1 \leq t \leq 1$

a) $v(t) = x'(t) = \cos(\pi t^2) * 2\pi t$

b) $a(t) = x''(t)$

$$v = 2\pi t$$

$$v' = 2\pi$$

$$v = \cos(\pi t^2)$$

$$v' = -2\pi t \sin(\pi t^2)$$

$$= -4\pi^2 t^2 \sin(\pi t^2) + 2\pi \cos(\pi t^2)$$

c) when $v(t) = 0$

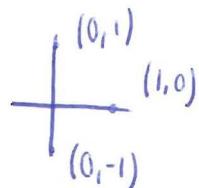
$$2\pi t \cos(\pi t^2) = 0$$

$$2\pi t = 0$$

$$t = 0$$

$$\cos(\pi t^2) = 0$$

$$t = -.707, .707$$



d) $(-.707, 0) \vee (.707, 1)$

$v(t)$ is negative or below x-axis

③ $t \geq 0$ $a(t) = 12t^2 - 4$

a) velocity = 0

$$\int 12t^2 - 4 dt = 4t^3 - 4t + C$$

$$v(t) = 4t^3 - 4t = 0$$

$$C=0$$

$$4t(t^2 - 1) = 0$$

$$t=0, t=1, \cancel{t=-1}$$

b) $\int v(t) = t^4 - 2t^2 + C$

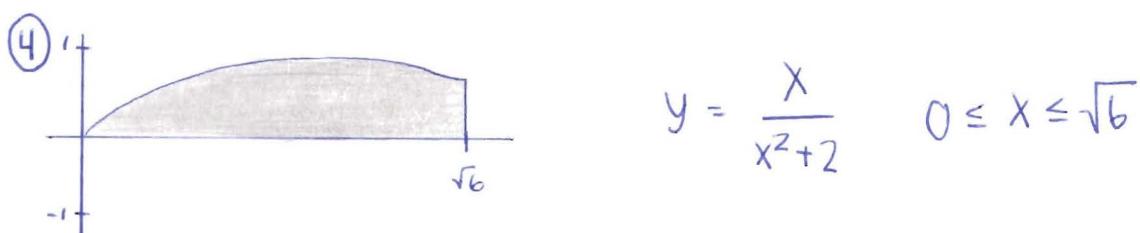
$$3 = 1^4 - 2(1)^2 + C$$

$$3 = -1 + C$$

$$C = 4$$

$$x(t) = t^4 - 2t^2 + 4$$

c) $\int_0^2 |v(t)| dt = 10 \text{ units}$



a) $\int_0^{\sqrt{6}} \frac{x}{x^2+2} - 0 dx = \ln|x| \cdot u^2$

b) $\int_0^K \frac{x}{x^2+2} dx = \frac{1}{2} \ln|2|$

$$u = x^2 + 2$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

$$\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|x^2+2| \Big|_0^K$$

$$\frac{1}{2} \ln|2| = \frac{1}{2} \ln|K^2+2| - \frac{1}{2} \ln|2| \Rightarrow \ln|2| = \frac{1}{2} \ln|K^2+2|$$

$$\ln|K^2+2| = \ln(4)$$

$$K^2+2 = 4$$

$$K = \pm\sqrt{2}$$

$$⑩ \quad g(x) = \int_1^x f(t) dt$$

$$\text{a) } g(4) = \int_1^4 f(t) dt$$

$$= \frac{1}{2}(1)(4+1)$$

$$+ \frac{1}{2}(1)(1)$$

$$- \frac{1}{2}(1)(1)$$

$$= \frac{5}{2} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}$$

$$= 5/2 \text{ u}^2$$

$$g(-2) = \int_1^{-2} f(t) dt = - \int_{-2}^1 f(t) dt$$

$$= - \left[\frac{1}{2}(3)(4) \right]$$

$$= -6 \text{ u}^2$$

b) derivative at $x=1$
 ↴ slope

$$g(x) = \int_1^x f(t) dt$$

$$g'(x) = f(x)$$

$$g'(1) = f(1) = 4$$

c) critical points

$x=3$
 ↑
 max
 (above to
 below)

endpoints

$x=-2$
 $x=4$ } minimums

$[-2, 3] f(x)$ is increasing

$[3, 4] f(x)$ is decreasing

There is more + area under
 the curve than -. So that
 means $f(-2) < f(3)$

abs min at $x=-2$

$$\text{d) } g'(x) = f(x)$$

$g''(x) = f'(x) \rightarrow$ switch from + to - or - to +.

Point of inflection is only at $x=1$

d) when $g''(x)$ switches from $>0 \rightarrow <0$ or <0 to >0

$$g(x) = \int_0^x f(t) dt$$

$$g'(x) = f(x)$$

when $f'(x) = 0$ or $f'(x) = \text{DNE}$

$$g''(x) = f'(x)$$

$$x=0 \quad x=3$$

⑨ d) P01 $f''(x) = 0$

slope of $f'(x)$ switches from + to - or - to +.

$$x=1 \quad x=3$$

b) abs. minimum $f'(x) = 0$ (below \rightarrow above)

critical pts.

$$x=4$$

endpts

$$x=-1$$

$$x=5$$

$f(x)$ decreases from $[-1, 4]$
" increases from $[3, 5]$

abs. min. at $x=4$

abs max at $x=-1 \rightarrow$ there is more - distance traveled
than +. So $f(-1) > f(5)$

c) $g(x) = x f(x)$

Point

$$x=2$$

$$y = 2(f(2))$$

$$= 2 \cdot 6$$

$$= 12$$

$$(2, 12)$$

Slope

$$u = x \quad u' = 1$$

$$v = f(x) \quad v' = f'(x)$$

$$x f'(x) + f(x) \Big|_{x=2}$$

$$y - 12 = 4(x-2)$$

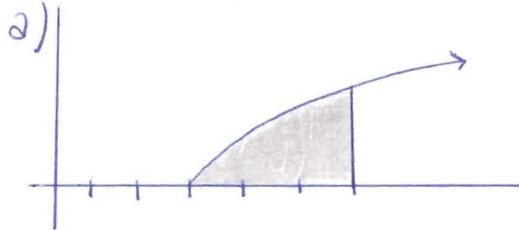
$$= 2f'(2) + f(2)$$

$$= 2(-1) + 6$$

$$= 4$$

$$c) \frac{1}{\sqrt{6}-0} \int_0^{\sqrt{6}} \frac{x}{x^2+2} dx = \frac{1}{\sqrt{6}} \cdot \ln|2| \cdot u^2 = .283 u^2$$

⑤ $f(x) = \sqrt{x-3}$



c) $A(w) = \int_3^w \sqrt{x-3} dx$

b)

$$\int_3^6 \sqrt{x-3} dx$$

$$\int_0^3 u^{1/2} du$$

$$u = x - 3$$

$$du = dx$$

$$x = 3 \rightarrow u = 0$$

$$x = 6 \rightarrow u = 3$$

$$= \frac{2}{3} u^{3/2} \Big|_0^3$$

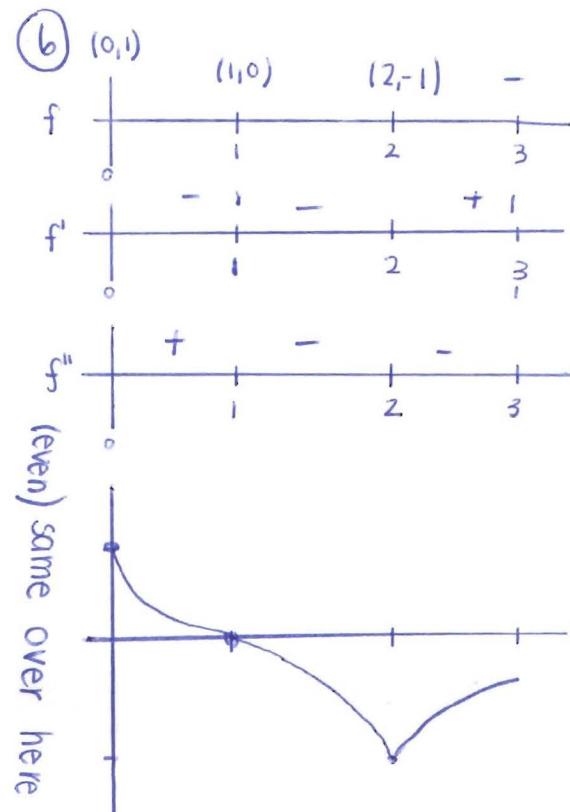
$$= \frac{2}{3} (\sqrt{3})^3 u^2$$

$$= 2\sqrt{3} u^2$$

d) rate of change \rightarrow derivative

$$A(w) = \int_3^w \sqrt{x-3} dx$$

$$\frac{dA}{dw} = \frac{d}{dx} \int_3^w \sqrt{x-3} dx = \sqrt{w-3} \Big|_{w=6} = \sqrt{3}$$



()

a) abs. max. \hat{o} $x=0$
abs. min. \hat{o} $x=2, -2$

b) P.O.I. $\rightarrow f''(x) > 0 \rightarrow < 0$ or
 $f''(x) < 0 \rightarrow > 0$ $x=1, -1$

c) see left

7) a) $f'(x) = 0 \rightarrow x = -7, -1, 4, 8$

b) max $\rightarrow f'(x) > 0 \rightarrow f'(x) < 0$
 $x = -1, 8$

c) $f''(x) < 0 \rightarrow$ slope of $f'(x)$ is negative
 $-3 < x < 2 \quad \frac{1}{2} \quad 6 < x < 10$

8) $g(x) = \int_0^x f(t) dt$

a) $g(3) = \int_0^3 f(t) dt$

$$= \frac{\pi r^2}{4} + \frac{1}{2}bh = \pi - \frac{1}{2}u^2$$

$$= \frac{\pi (2)^2}{4} + \frac{1}{2}(1)(-1)$$

b) relative max. when $g'(x) = 0$ or $g'(x) = \text{DNE}$ (positive to neg.)

$$\frac{d}{dx} \int_0^x f(t) dt = 0 \Rightarrow f(x) = 0 \quad x = 2, 4$$

$x = 2$

↑ below
above to
below above
(max) (min)

c) $g(3) = \pi - \frac{1}{2}$ $g'(3) =$

$$(3, \pi - \frac{1}{2}) \quad g'(x) = f(x)$$

$$g'(3) = f(3)$$

$$f(3) = -1$$

$\underbrace{}$
slope

$$Y - (\pi - \frac{1}{2}) = -1(x - 3)$$