Calculators are allowed on all problems.

2002 AB 2 (Form B)

1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$.

   (a) Find the area of region R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

1996 AB 2

2. Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

   (a) Find the area of region R.

   (b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

   (c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

2004 AB 1 (Form B)

3. Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the x-axis.

   (a) Find the area of region R.

   (b) Find the volume of a solid generated when R is revolved about the horizontal line $y = 3$.

   (c) Find the volume of a solid generated when R is revolved about the vertical line $x = 10$.

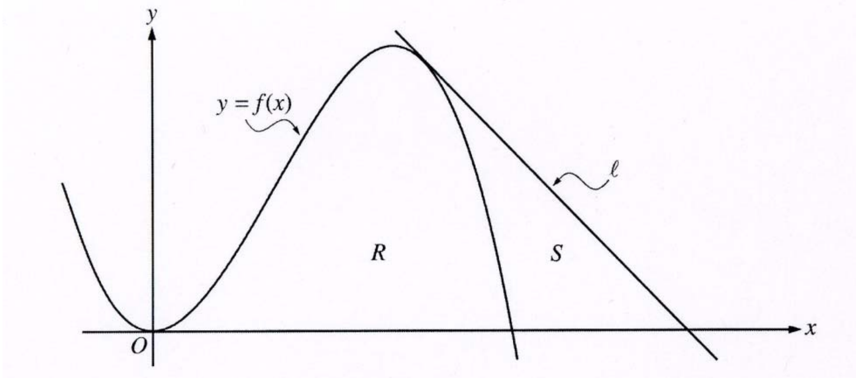
2003 AB 1 (Form B)

4. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is the tangent line to the graph of f. Let R be the region bounded by the x-axis and the graph of f, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown below.

(a) Show that line ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of region S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.



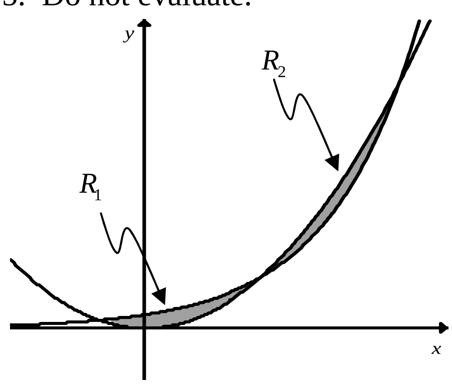
1995 AB 4

5. The shaded regions R_1 and R_2 shown below are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

(a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

(c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving region R_1 about the line $y = 5$. Do not evaluate.



Note: Figure not drawn to scale.

1991 AB 1 No Calculator

6. Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

(a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.

(b) Write an expression for $f(x)$.

(c) Find the average value of f on the interval $1 \le x \le 3$.

1989 AB 4 No Calculator

7. Let f be the function defined by $f(x) = \dfrac{x}{\sqrt{x^2 - 4}}$.

(a) Find the domain of f.

(b) Write an equation for each vertical asymptote to the graph of f.

(c) Write an equation for each horizontal asymptote to the graph of f.

(d) Find $f'(x)$.

1995 AB 1 Calculator allowed

8. Let f be the function given by $f(x) = \dfrac{2x}{\sqrt{x^2 + x + 1}}$.

(a) Find the domain of f. Justify your answer.

(b) In the viewing window [-5, 5] x [-3, 3], sketch the graph of f.

(c) Write an equation for each horizontal asymptote of the graph of f.

(d) Find the range of f. Use $f'(x)$ to justify your answer. Note: $f'(x) = \dfrac{x+2}{\left(x^2 + x + 1\right)^{\frac{3}{2}}}$.

1995 AB 3 Calculator allowed

9. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\dfrac{dy}{dx}$.

(b) Write an equation for the line tangent the curve at the point (4, -1).

(c) There is a number *k* so that the point (4.2, *k*) is on the curve. Using the tangent line found in part (b), approximate the value of *k*.

(d) Write an equation that can be solved to find the actual value of *k* so that the point (4.2, *k*) is on the curve.

(e) Solve the equation found in part (d) for the value of *k*.

1994 AB 6

10. Let $F(x) = \int_0^x \sin(t^2)\,dt$ for $0 \le x \le 3$.

(a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0, 1] to approximate *F*(1).

(b) On what intervals is *F* increasing?

(c) If the average rate of change of *F* on the closed interval [1, 3] is *k*, find $\int_1^3 \sin(t^2)\,dt$ in terms of *k*.