Algebra 2

Summer Assignment 2012

This assignment will be due the first day of school, and a test will be given on this material the second week of school. Brief notes and explanations are included within each section. If additional information is needed, the following websites may be helpful:

http://www.regentsprep.org/Regents/math/ALGEBRA/FormulaSheetAlgebra.pdf

http://regentsprep.org/

http://www.purplemath.com/modules/index.htm

http://www.math.com/students/practice.html

http://www.algebra-class.com

http://www.themathpage.com/alg/algebra.htm

http://www.sosmath.com/algebra/algebra.html

Simplifying and Evaluating Algebraic Expressions

Order of Operations

1.) First, do operations that occur within grouping symbols.

2.) Next, evaluate powers.

3.) Then, do multiplication and divisions from left to right.

4.) Finally, do additions and subtractions from left to right.

In 1 – 2, evaluate each expression. SHOW ALL WORK!!

1.) $(25 \div 5 + 3)^2 \div 16$

2.) $3 \cdot 6 - 4 \cdot 5$

In 3 – 6, evaluate the following expressions given that a = 2, b = -2 and c = -3. SHOW ALL WORK!!

3.) $abc - a(b - c)$

4.) $-a^2 - b^2 - 2c$

5.) $\dfrac{\sqrt{2a} - 2b - c}{|abc|}$

6.) $\dfrac{a^2b^2 + 2}{a^2b^2 - 1}$

Solving Linear Equations

<u>Your goal is to isolate the variable on one side of the equation</u>.

For example:

$\frac{3}{7}x + 9 = 15$ ***Original equation***

$\frac{3}{7}x = 6$ ***Subtract 9 from each side.***

$x = \frac{7}{3}(6)$ ***Multiply each side by the reciprocal.***

$x = 14$ ***Simplify***

Solve the following equations for x:

7.) $x - 3 = 4x + 15$

8.) $-x + 3 = 7x + 8$

9.) $5(3 - 4x) = 7 - (4 - x)$

10.) $7x + 14 - 3x = 4x + 14$

11.) $2 - \frac{1}{2}x = 4 - \frac{1}{4}x$

12.) $12x - (6 + 8x) = \frac{1}{2}(8x - 12)$

13.) $8x - 9 - 2x = 2(3x - 1) + 7$

14.) $\frac{2}{3}x + 5 = \frac{3}{5}$

Rewriting Equations and Formulas

Example 1: Rewriting an equation with more than one variable

Solve $7x - 3y = 8$ for y

Steps: $7x - 3y = 8$

$-3y = 8 - 7x$ ***subtract 7x from each side***

$y = -\frac{8}{3} + \frac{7}{3}x$ ***divide each side by -3***

***Solve for y in terms of x*:**

1. $4x + 8y = 17$
2. $\frac{3}{4}x + 5y = 20$
3. $xy + 2x = 8$
4. $\frac{2}{3}x - \frac{1}{2}y = 12$

Example 2: Rewriting a common formula

Solve $P = 2l + 2w$ for w

Steps: $P = 2l + 2w$

$P - 2l = 2w$ ***subtract 2l from each side***

$\frac{P}{2} - l = w$ ***divide each side by 2***

Solve the formula for the indicated variable:

5. $A = \frac{1}{2}bh$; solve for b

6. $A = \frac{\pi r^2 S}{360}$; solve for S

7. $N = 3a^2 b$; solve for b

8. $A = \frac{1}{2}(b_1 + b_2)h$; solve for h

Solving Linear Inequalities

Solving a linear inequality is a lot like solving a linear equation with **one important exception.** When **multiplying** or **dividing** both sides **BY A NEGATIVE NUMBER**, you must **REVERSE THE INEQUALITY SYMBOL**.

Ex: $-2x < 1$ becomes $x > -\frac{1}{2}$

Ex: $2x < -1$ becomes $x < -\frac{1}{2}$

Ex: $\dfrac{-x}{2} \geq 1$ becomes $x \leq -2$

Ex: $\dfrac{x}{2} > -1$ becomes $x > -2$

Compound Linear Inequalities:

Ex: $-2 \leq -2y + 4 \leq 14$

$-6 \leq -2y \leq 10$ …after subtracting 4 from **all three sides**

$3 \geq y \geq -5$ …after dividing **all three sides** by –2 (notice that the symbols reversed)

$-5 \leq y \leq 3$ …after reversing the entire inequality. This last step is not required, but is "cleaner."

This translates to "-5 is less than or equal to *y* which is less than or equal to –1."
This also translates to "-5 is less than or equal to *y* AND *y* is less than or equal to –1."

Solve the inequality and then graph your solution.

1. $3x + 5 < 20$

2. $5 - x < 9$

3. $7x - 3 \geq 13 + 3x$

4. $-y + 5 \leq 3y - 3$

5. $3 < x + 2 < 9$

6. $-2 < 2x + 4 < 10$

7. $x + 3 \leq 5$ or $x - 2 \geq 5$

8. $2x + 1 < -7$ or $3x - 4 > 2$

Linear Equations and Functions

Graphing Functions and Relations

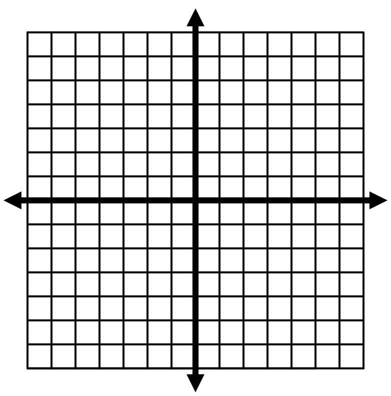
- Recall that for each input or x – value, there is exactly one output or y – value in order for a ***relation*** to be classified as a ***function***.
- Plot the points (x, y) listed below on a coordinate grid and use the ***vertical line test*** to determine whether the relation is a function.

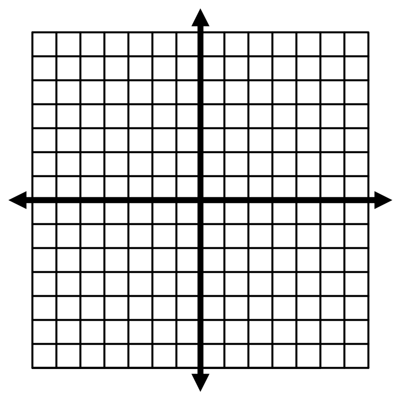
1.

x	-2	-1	0	1	2
y	-1	-1	0	1	1

2.

x	-1	-1	-1	0	1
y	-1	0	1	1	1





- Evaluate the function for the given value of x:

3. $f(x) = -x + 3$ when $x = -2$

4. $f(x) = -5 + 8x^2$ when $x = \frac{1}{2}$

5. $f(x) = |x+3| - 9$ when $x = -4$

6. $f(x) = 2x^3 - 7x^2 + 8$ when $x = -3$

- Recall the formula for the ***slope*** of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Find the slope of each line going through the given points. Then determine which lines are ***parallel*** or ***perpendicular***.
- Recall that parallel lines have the same slope and perpendicular lines have slope that are opposite reciprocals (ex: m = ½ and m = – 2)

A. (3, 4) and (1, 6)

B. (– 1, 0) and (3, 5)

C. (1, 5) and (– 4, – 5)

D. (– 1 – 9) and (2, – 3)

E. (– 6, 7) and (– 3, 6)

F. (– 1, – 9) and (1, – 3)

- Recall the different forms of a linear function:
  - Slope – intercept: $y = mx + b$, where m= *slope* and b = y – int
  - Standard form: $ax + by = c$, where a, b = coefficients and c = constant
  - Point – slope form: $y - y_1 = m(x - x_1)$, where m = slope and (x_1, y_1) is a point on the line

- Determine the slope and y – intercept of the equation of the line (Hint: rewrite the equation into $y = mx + b$ form).

7. $y = 2x + 10$ $m =$ $b =$

8. $y = 2x$ $m =$ $b =$

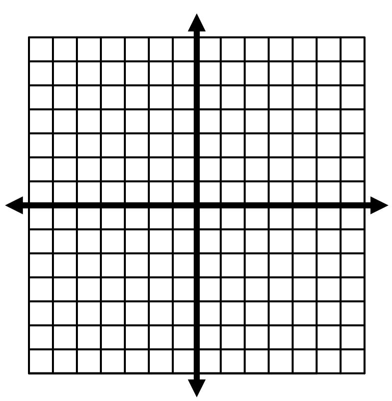
9. $x = -1$ $m =$ $b =$

10. $y = 5$ $m =$ $b =$

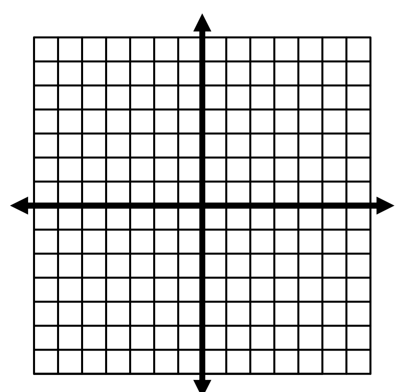
11. $5x - y = 12$ $m =$ $b =$

- Given the slope and y – intercept, draw the line on the graph provided.

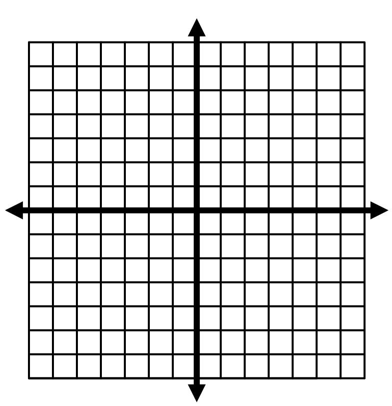
12. $m = 2$, $b = -4$



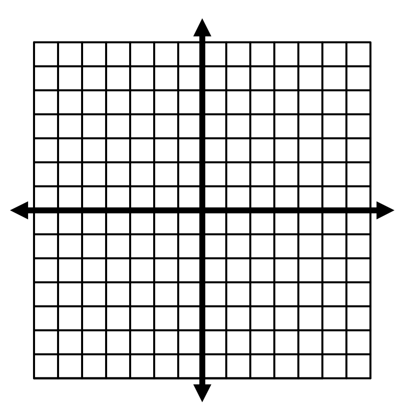
13. $m = 0$, $b = 4$



14. $m = ½$, $b = 2$

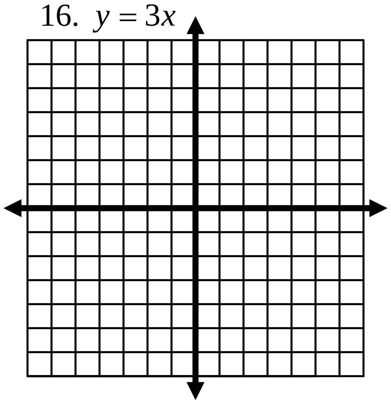


15. $m = -\frac{4}{5}$, $b = -1$

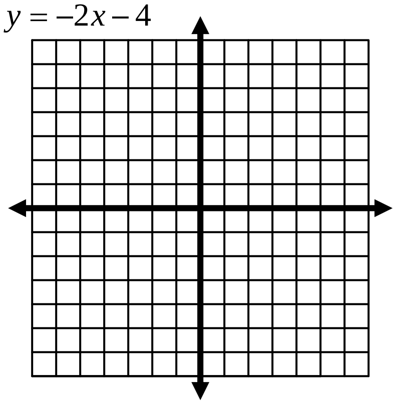


- Graph the equation of the line using any method.

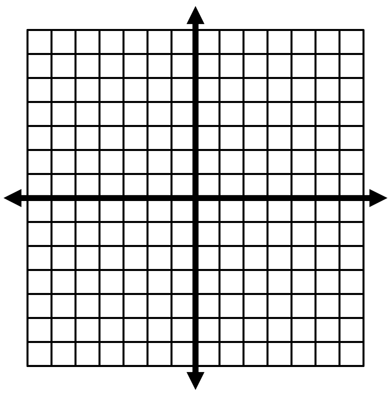
16. $y = 3x$



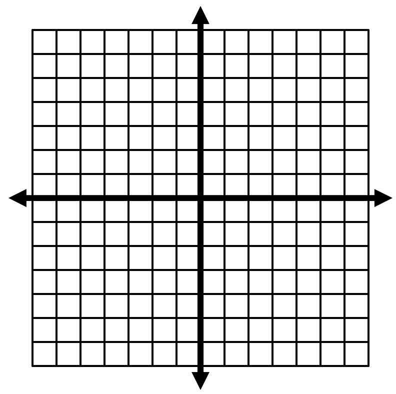
17. $y = -2x - 4$



18. $x + 2y = 1$



19. $x = -5$



- Write the equation of the line given the following:
    - Hint: You may use slope – intercept form ($y = mx + b$) or point – slope form ($y - y_1 = m(x - x_1)$) to write the equation

20. Slope = 2, y – intercept = – 4   20.______________________

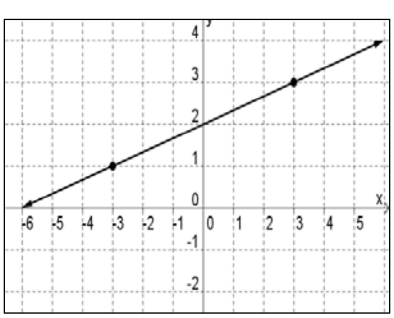
21. Slope = 0, y – intercept = 2   21.______________________

22. Slope = 2, passes through (1, - 3)   22.______________________

23. Passes through (1, -6) and (4, -3)   23.______________________

24. Passes through (2, 6) and (-7, 6)   24.______________________

25.

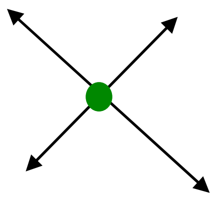


25. ______________________

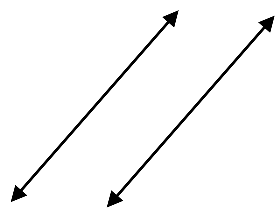
Solving Systems of Equations by Graphing

- **To "solve a system of equations" means to find the point of intersection.** (We are working with linear equations, therefore, we are talking about the point of intersection of two lines.)

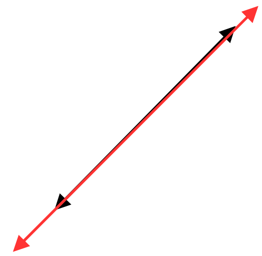
- **There are three possible answers due to the relationships that the lines may have:**



They may intersect at exactly one point. (Therefore, the solution is that point.)



The lines are parallel and never intersect.($\therefore \varnothing$)

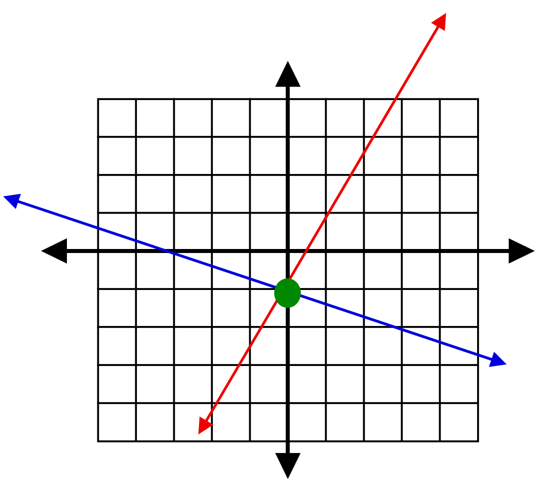


The lines are "coincident," meaning they overlap. ($\therefore$ solution is "All points on the line.")

- **To solve a system of equations by graphing, simply graph the lines on the same coordinate plane, and state the solution.**

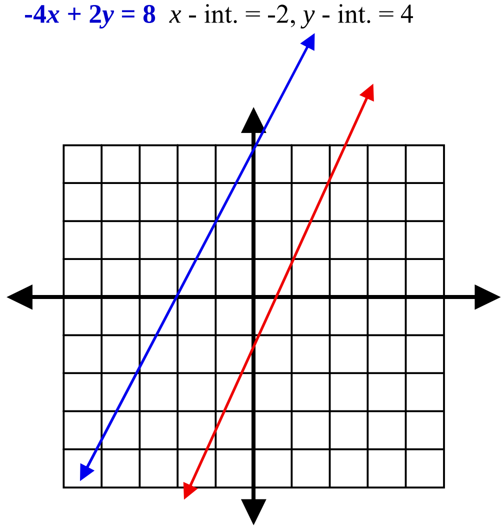
- Examples:

1) $\mathbf{y = 2x - 1}$ $\quad m = 2, b = -1$
$\mathbf{x + 3y = -3}$ $\quad x$ - int. = -3, y - int. = -1



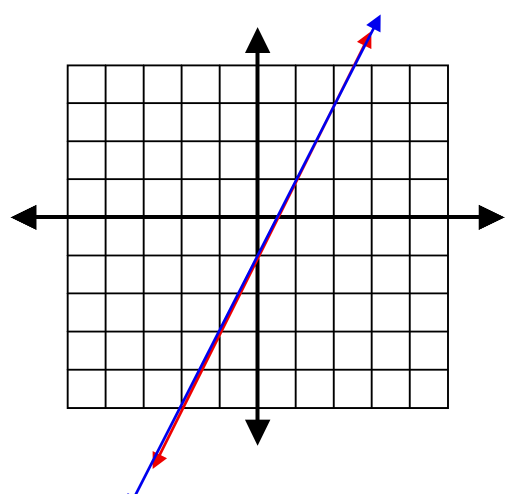
Solution: (0, -1)

2) $\mathbf{y = 2x - 1}$ $\quad m = 2, b = -1$
$\mathbf{-4x + 2y = 8}$ $\quad x$ - int. = -2, y - int. = 4



These lines are parallel, and therefore, will never intersect.
Solution: No solutions (or $\varnothing$)

3) $\mathbf{y = 2x - 1}$ $\quad m = 2, b = -1$
$\mathbf{4x - 2y = 2}$ $\quad x$ - int. = $\frac{1}{2}$, y - int. = -1

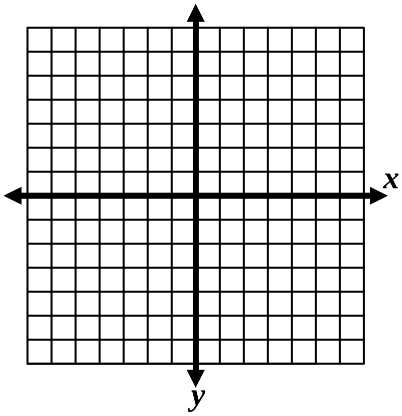


These lines are "coincident," and therefore, will always intersect.
Solution: All points on the line.

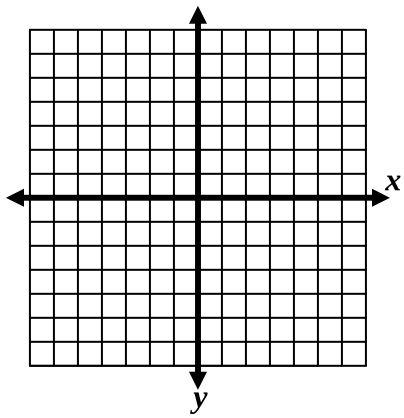
Now try these on your own.

Solve each system of equations by graphing.

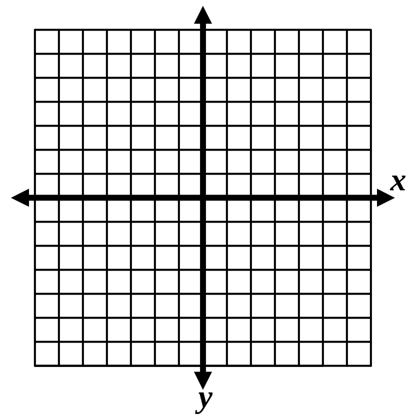
1. $y = -x + 6$
   $y = x - 2$



2. $2x + y = 3$
   $4x + 2y = 6$



3. $x + 2y = 0$
   $y + 3 = -x$



Solving Systems of Equations Algebraically

SUBSTITUTION

EXAMPLE: $\mathbf{y = 3x - 2}$
$\mathbf{2x + 3y = 5}$

*NOTICE: The first equation is solved for *y*. This makes it really easy to use substitution.

$2x + 3y = 5$
$2x + 3\mathbf{(3x - 2)} = 5$

*Replace '*y*' in the second equation with the expression from the first equation.

$2x + 9x - 6 = 5$
$11x - 6 = 5$
$11x = 11$
$x = 1$

*You are now left with one variable. Solve the equation.

*REMEMBER: To solve a system of equations, you must find the point of intersection. We have our *x*-coordinate; now we must find our *y*-coordinate. Plug in the value you know into one of your equations to find your other value.

$y = 3x - 2$
$y = 3\mathbf{(1)} - 2$
$y = 3 - 2$
$y = 1$

THEREFORE, OUR SOLUTION IS (1, 1)

ELIMINATION

EXAMPLE: $\boldsymbol{x - y = 2}$
$\boldsymbol{x + y = 4}$

*NOTICE: The coefficients of y are exact opposites. This makes it really easy to use elimination.

$$\begin{array}{r} x - y = 2 \\ \underline{+\,x + y = 4} \\ \frac{2x}{2} = \frac{6}{2} \end{array}$$

*Combine your equations.
*One variable is eliminated. Solve for the remaining variable.

$x = 3$
$x + y = 4$
$3 + y = 4$
$y = 1$

*Use this value to find the other member of the ordered pair. Substitute into either of your original equations.

THEREFORE, OUR SOLUTION IS (3, 1)

EXAMPLE: $\boldsymbol{2x - 2y = 4}$
$\boldsymbol{x + y = 2}$

*NOTICE: you do not have a pair of exact opposites between your two equations. Therefore, you must multiply one (or both) of your equations by something that will enable elimination.

$2x - 2y = 4$ --------> $2x - 2y = 4$
$2(x + y = 2)$ --------> $2x + 2y = 4$

NOW, combine and eliminate a variable. $4x = 8$

Solve for remaining variable. $x = 2$

Use this value to find the other member of your ordered pair. Use any equation.

$x + y = 2$
$2 + y = 2$
$y = 0$

Write your solution: **(2, 0)**

Now try these on your own.

In 1 - 3, solve each system using substitution. SHOW ALL WORK!!

1. $x + y = 7$
   $x = y + 9$

2. $y = 2x + 32$
   $2x + y = 60$

3. $x + y = 3$
   $2x + 2y = -4$

In 4 – 6, solve each system using elimination. SHOW ALL WORK!!

4. $8x - 2y = 8$
   $x + 2y = 3$

5. $3y - 8x = 9$
   $y - x = 2$

6. $x + 2y = -1$
   $2x + 4y = -2$

In 7 – 8, define your variables and write a system of equations to represent each scenario described. Then solve your system using any method. SHOW ALL WORK!!

7. A class of 195 students went on a field trip. They took 7 vehicles, some cars and some buses. Find the number of cars and the number of buses they took if each car holds 5 students and each bus hold 45 students.

8. There are 13 animals in the barn. Some are chickens and some are pigs. There are 40 legs in all. How many of each animal are there?

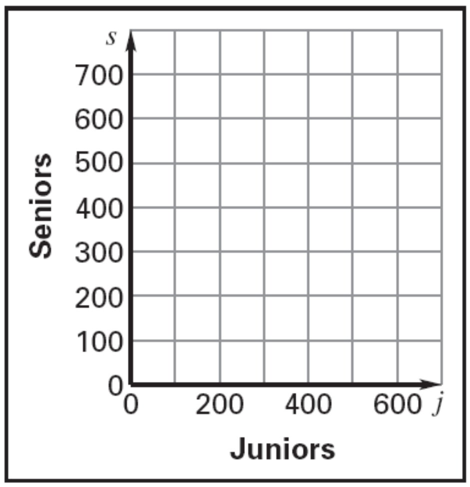
Use the information below for #9 – 12.

The junior class at Brady High School sponsors the prom. This year's junior class did not earn enough on fundraising activities to offset the cost, so the junior class members have agreed to pay a higher ticket price than the seniors.

Thus, junior tickets cost \$10.50 and senior tickets cost \$7.50. Planners expect 560 juniors and seniors to buy tickets. They need to collect \$5000 on ticket sales to cover the remaining expenses.

9. Let j represent the number of juniors and let s represent the number of seniors. Write a system of linear equations to model the situation.

10. Graph the system.



11. Is it easy or difficult to determine from the graph how many juniors and seniors need to attend the prom for the class to break even? Use your graph to approximate how many juniors and seniors need to attend the prom for the class to break even.

12. Use substitution or elimination find the exact number of juniors and seniors that need to attend the prom in order for the class to break even.